Polynomial Time Approximation Algorithms for Localization based on Unknown Signals

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Motivation

- Can we localize our stuff just from ambient sound?

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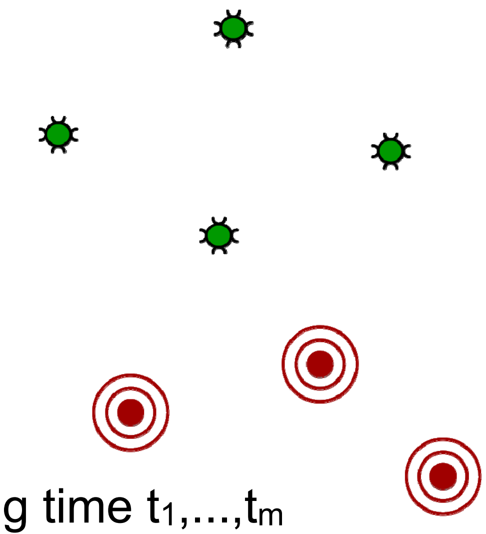
Localization without Anchors

[G. Doré, 1862]

Problem Setting

- n receiver nodes: $M_1, M_2, \ldots, M_n$
  - synchronized clocks
  - connected by a computer network
  - unknown positions in the plane
- m signals: $S_1, S_2, \ldots, S_m$
  - points in the plane
  - unknown positions, unknown starting time $t_1,\ldots,t_m$
  - distinguishable
  - signal S_j received at receiver M_i at time of arrival (ToA)
    - $T_{ij} = |M_i - S_j|/c + t_j$
  - signal speed c is a known constant
- ToAs can be deviated by some measurement inaccuracies



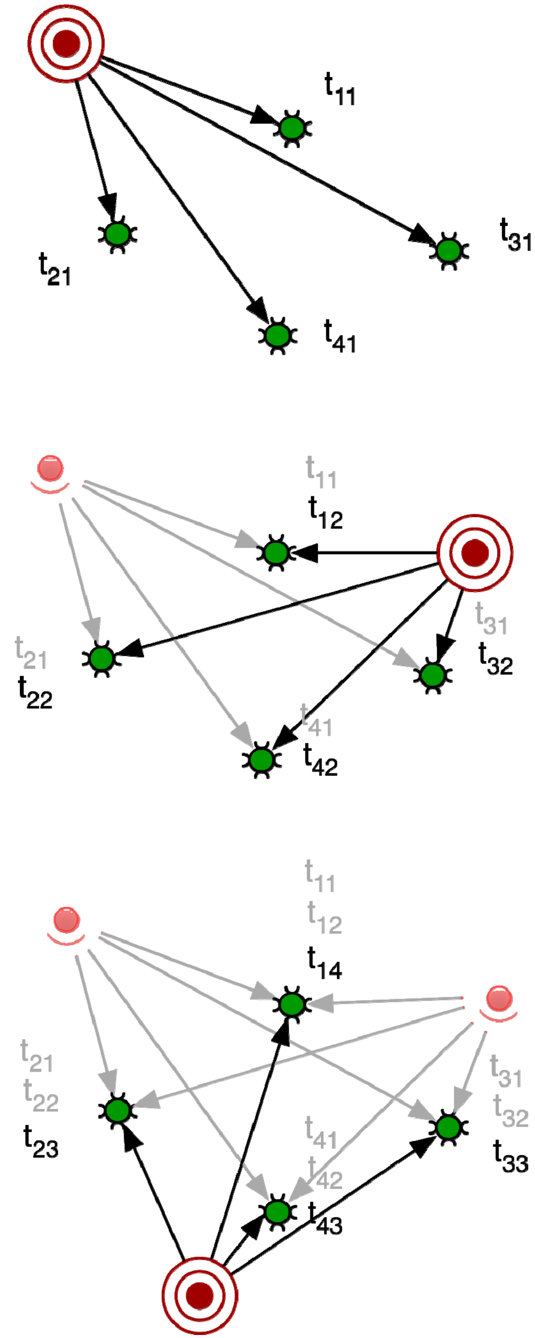
Problem Setting

- Sender S_j and receiver M_i ($j \le m$, $i \le n$) satisfy:

$$c\,(t_{M_i, S_j} - t_{S_j}) = \| M_i - S_j \|$$

with given: c Signal velocity
$t_{M,S}$ Signal reception time

and unknown: t_S Send time
S Sender positions
M Receiver positions



Applications

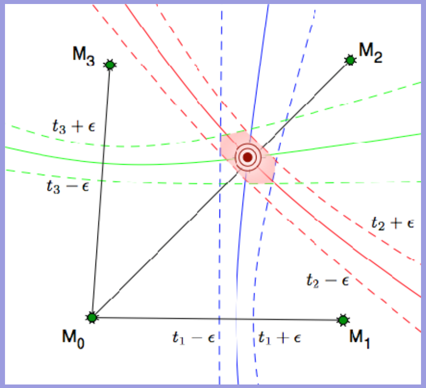
- Pool computer localization by clapping
- Localization of wireless sensor nodes by ambient noises
  - Cheap alternative to costly calibration
- Self-Configuring target localization
  - Bird finder
  - Cricket locator
  - User tracking in supermarkets / warehouses
- GPS using TV/Radio satellites
  - “Open Galileo”

Related Work for Localization without Anchors

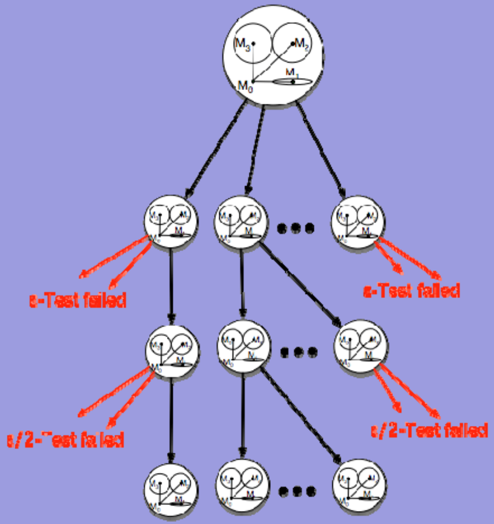
- **If the senders are distant**
  - Ellipsoid Method
    - Janson, Schindelhauer, Wendeberg: IPIN 2010, AlgoSensors 2010
  - Microphone Array
    - Valin et al. IROS 2003
  - Thrun, NIPS 2005
- **Many randomly distributed senders**
  - Schindelhauer, Lotker, Wendeberg, SIROCOO 2011
- **Iterative heuristics**
  - Biswas et al. IROS 2004,
  - Wendeberg, Höflinger, Schindelhauer, IPIN 2011
- **Closed-form solution for many receivers and senders**
  - Pollefeys, Nister, 2008

Two Main Components

- ε-Test discards obviously impossible receiver positions



- BFS tree search to explore all possible receiver positions



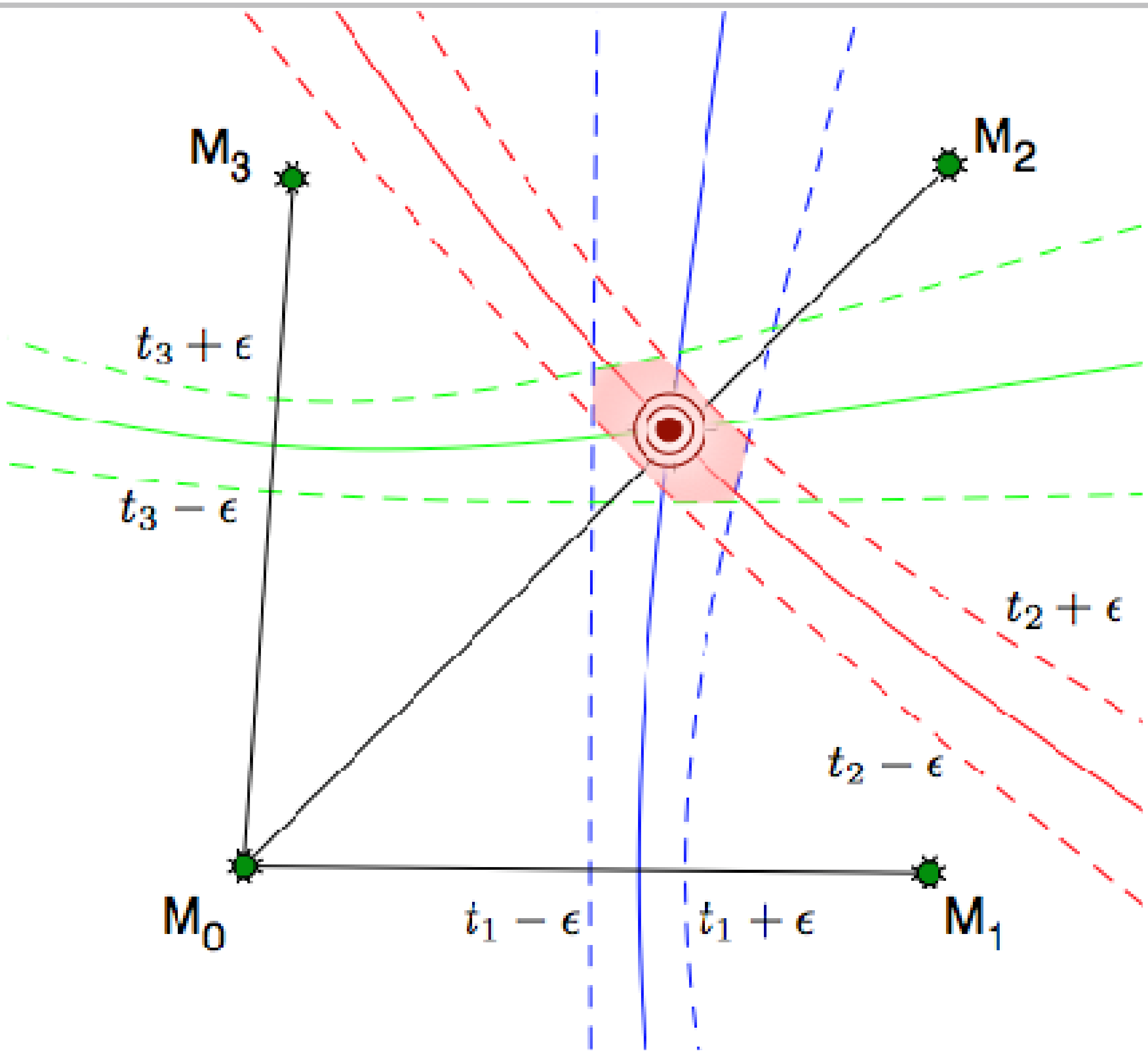
The ε-Test

- Idea
  - all measurements underly some accuracy $\varepsilon>0$
  - weaken the constraints by this term
- Definition ε-Test
  - a set of receivers M_i satisfies the ε-Test if there are signal positions S_j satisfying the following condition

$$-\varepsilon \leq c\,(t_{M_0, S_j} - t_{M_i, S_j}) - (\| S_j - M_0 \| - \| S_j - M_i \|) \leq +\varepsilon$$

- Motivation
  - the ε-Test gives some information about the stability of the solution
  - receivers satisfying the ε-Test may be in the vicinity of the correct solution
  - wrong positions of receivers can be easily discarded

New Approach: The ε-Test



Efficient computation of the ε-Test

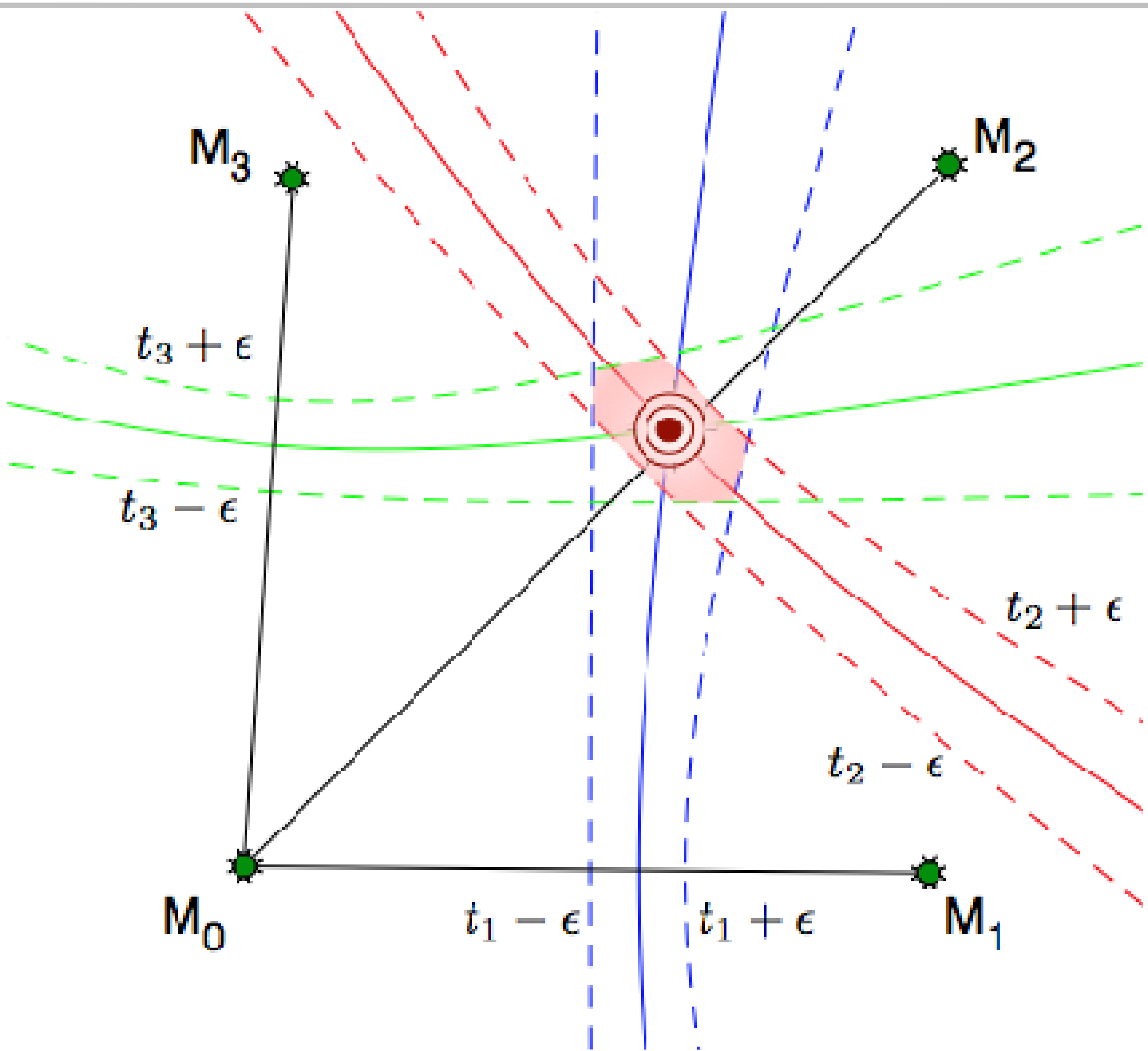
- For $M_0 = (0,0)$ compute all inequalities in polar coordinates

  $$c\,(t_{M0,\,Sj} - t_{Mi,\,Sj}) - (\| S_j - M_0 \| - \| S_j - M_i \|) \leq \varepsilon$$

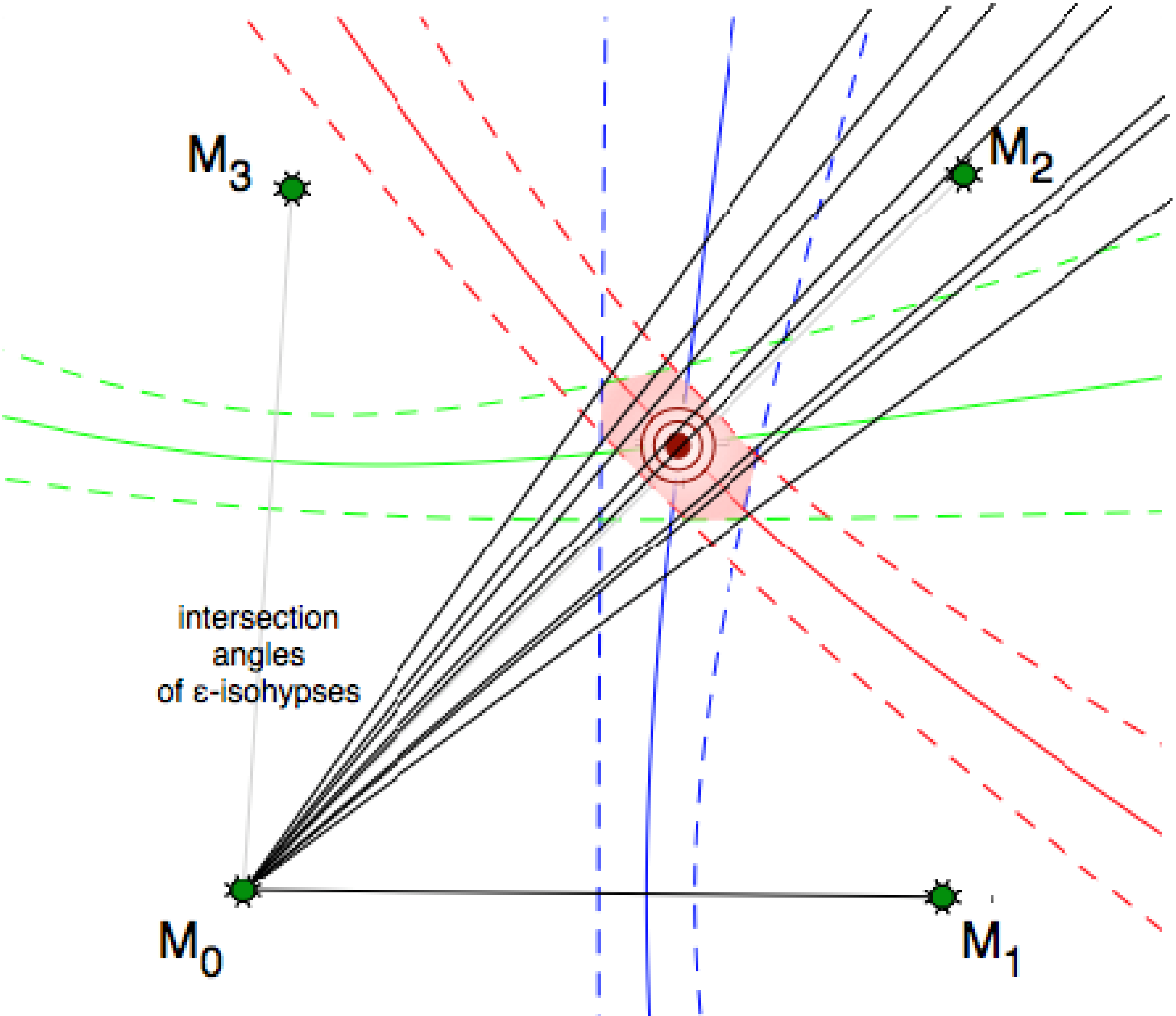
  $$c\,(t_{M0,\,Sj} - t_{Mi,\,Sj}) - (\| S_j - M_0 \| - \| S_j - M_i \|) \geq -\varepsilon$$

- Test whether the intersection of these sets is empty
  - for the test use a sweep-line algorithm in polar coordinates known from kinetic data structures
    - Compute all possible intersection angles
    - Sort the angles to get the sectors where no intersection occurs
    - For each sector test at an arbitrary position whether the maximum of all $-\varepsilon$-curves is smaller than the minimum of all $+\varepsilon$-curves.
- The test has run-time of $O(n^2\, m)$

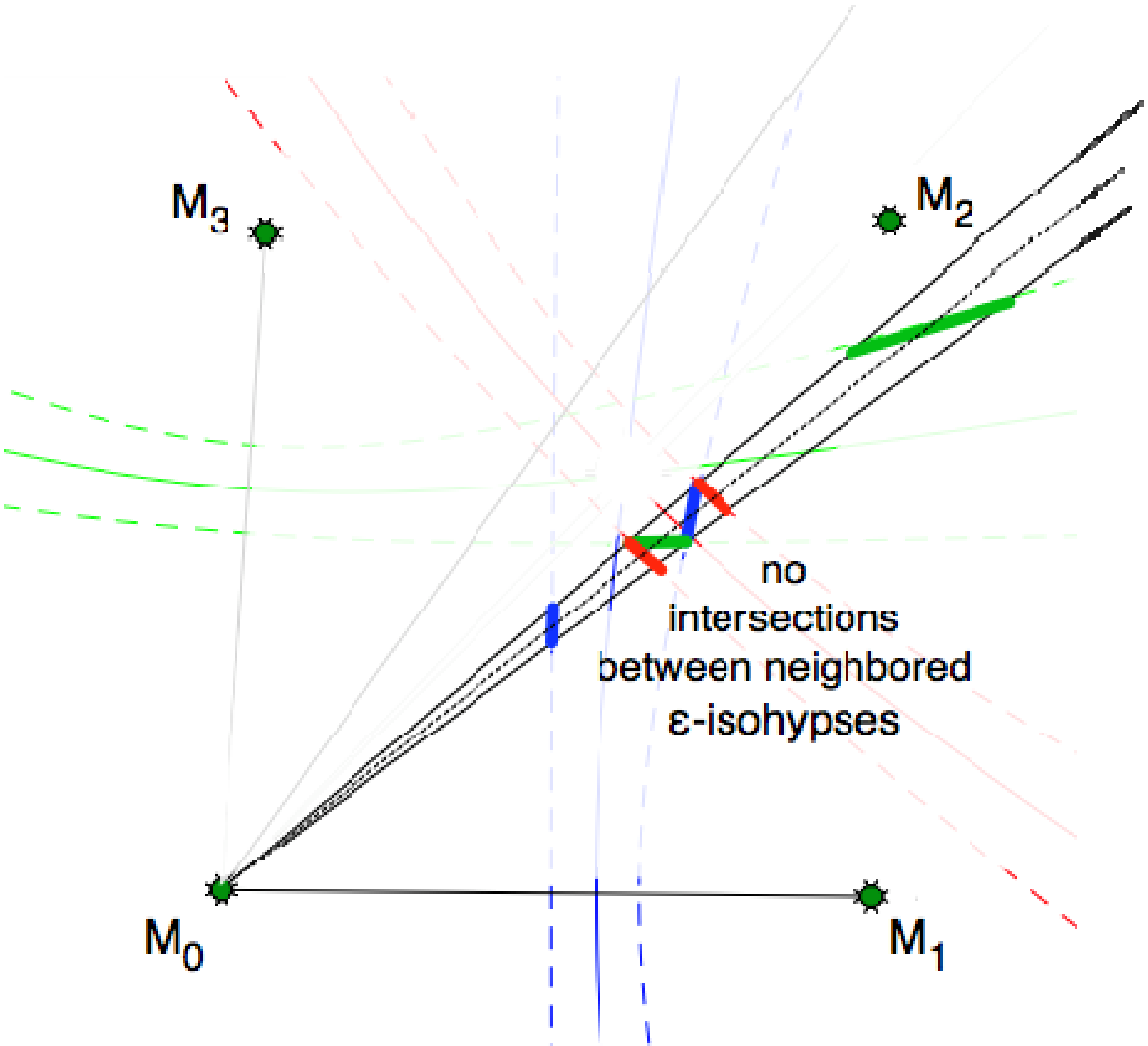
The ε-Test-Area



Compute all Possible Intersections



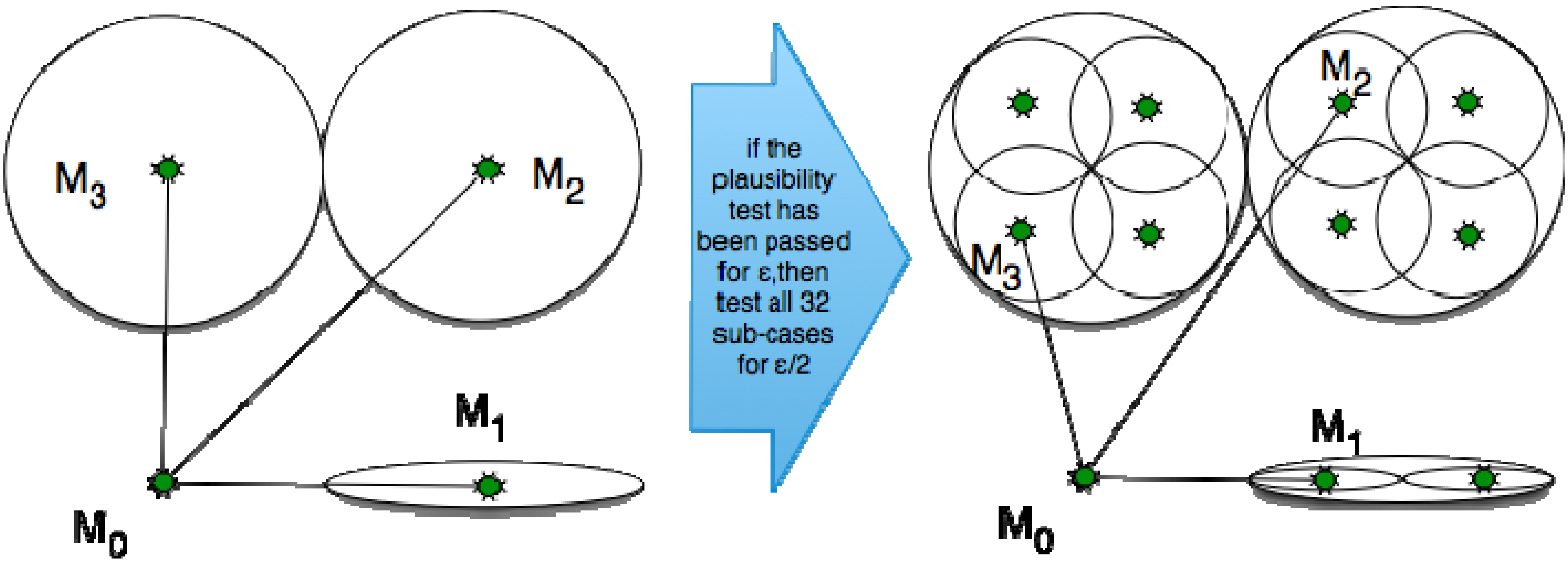
Test each Sector



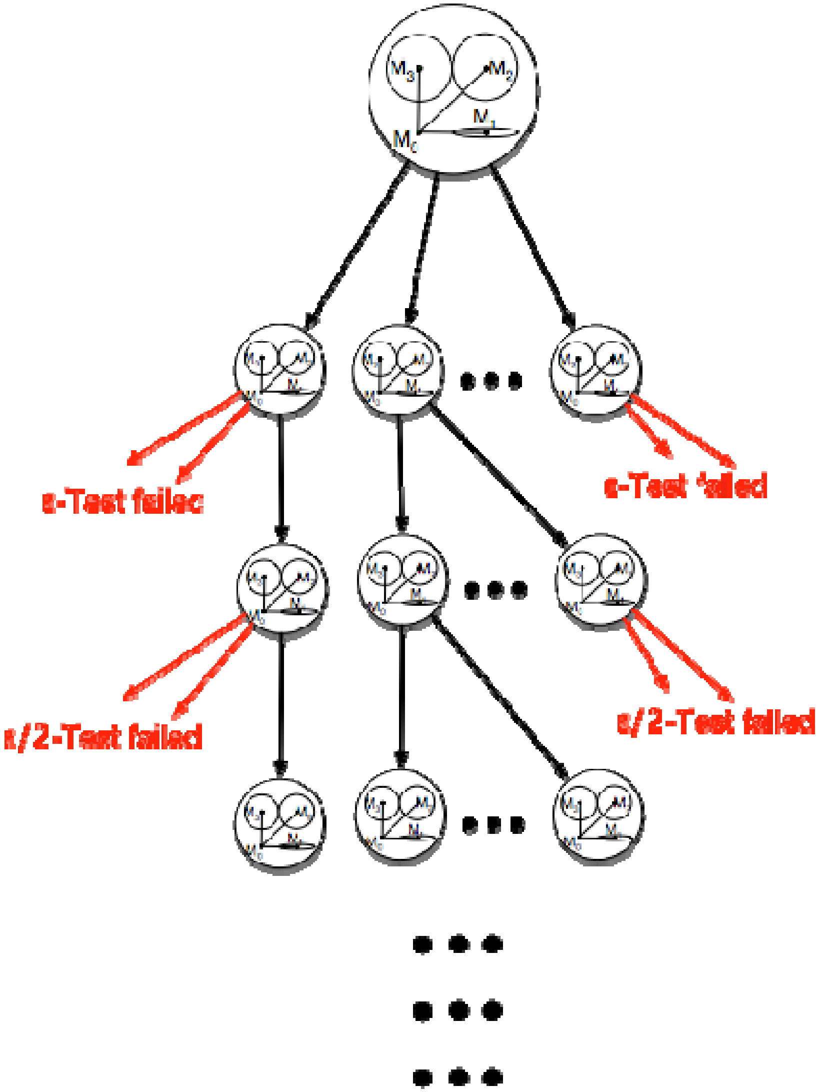
Properties of the ε-Test

- If a set of receivers succeeds in the ε-Test
  - then (by definition) there is a set of signals satisfying the constraints up to an inaccuracy of ε
- If a set of receivers fails in the ε-Test
  - then (by definition) there is no set of signals satisfying the constraints up to an inaccuracy of ε
  - then the set of receivers also fails in all ε'-Test with $\varepsilon' \leq \varepsilon$
- If a set of receivers succeeds in the ε-Test for all $\varepsilon>0$
  - then the set of receivers is an exact solution to the sound localization problem
- A set of receivers always succeeds in the D-Test,
  - where D is the maximum distance of all receivers

Refinement of the Grid



BFS for the Solution



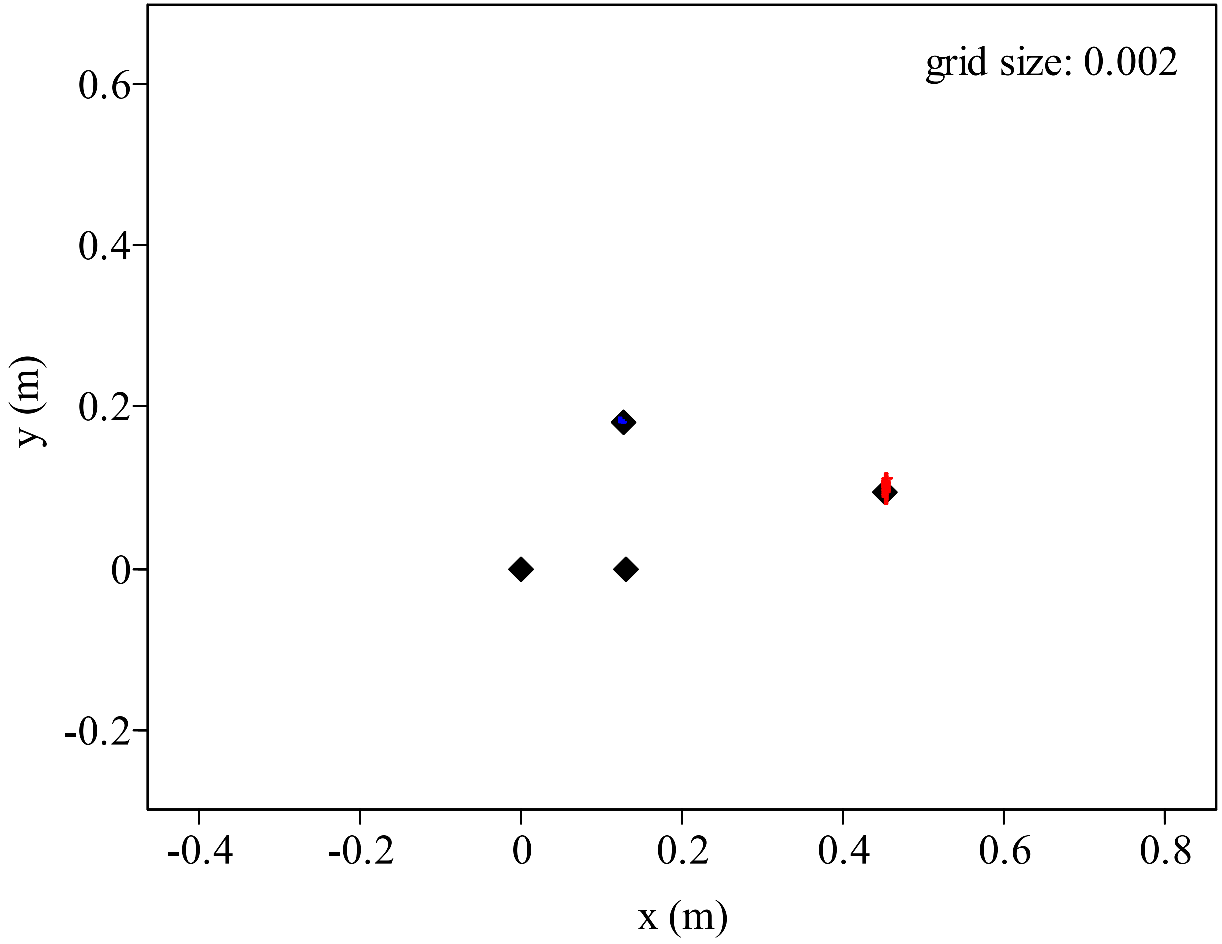
Efficient computation of the ε-Test

- **Theorem 1**
  The BFS and ε-Test solves the approximation problem of self-localization of n **receivers** in time
  $$\mathcal{O}((\sqrt{2}/\epsilon)^{2n-3}mn^2)\ .$$

- **Corollary**:
  For four receivers and m senders the runtime is
  $$\mathcal{O}(\epsilon^{-5}m).$$

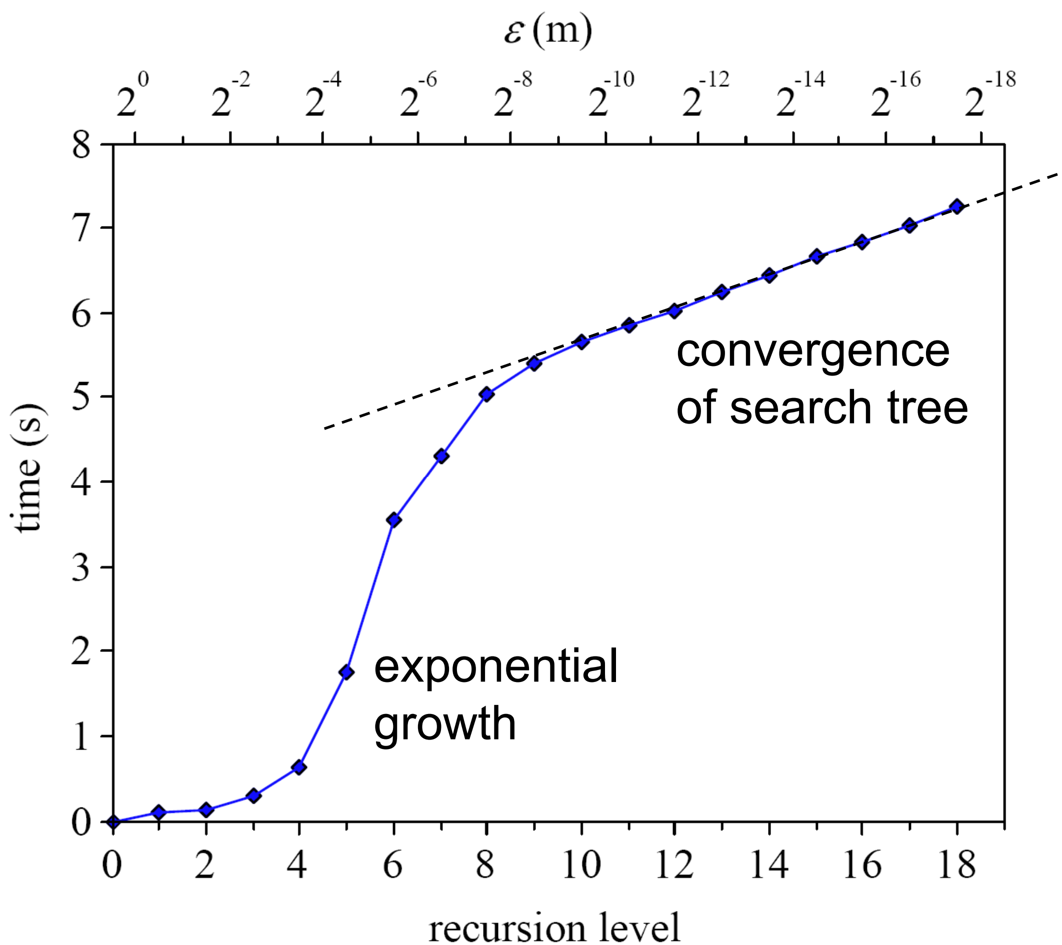
- **Theorem 2**
  The analogous problem for m **signals** can be solved in
  $$\mathcal{O}(\left(\sqrt{2}/\epsilon\right)^{2m-3} nm\log m).$$

Refinement for 4 Receivers



Experimental evaluation

- Runtime example (Intel Core-i5, 4 cores)

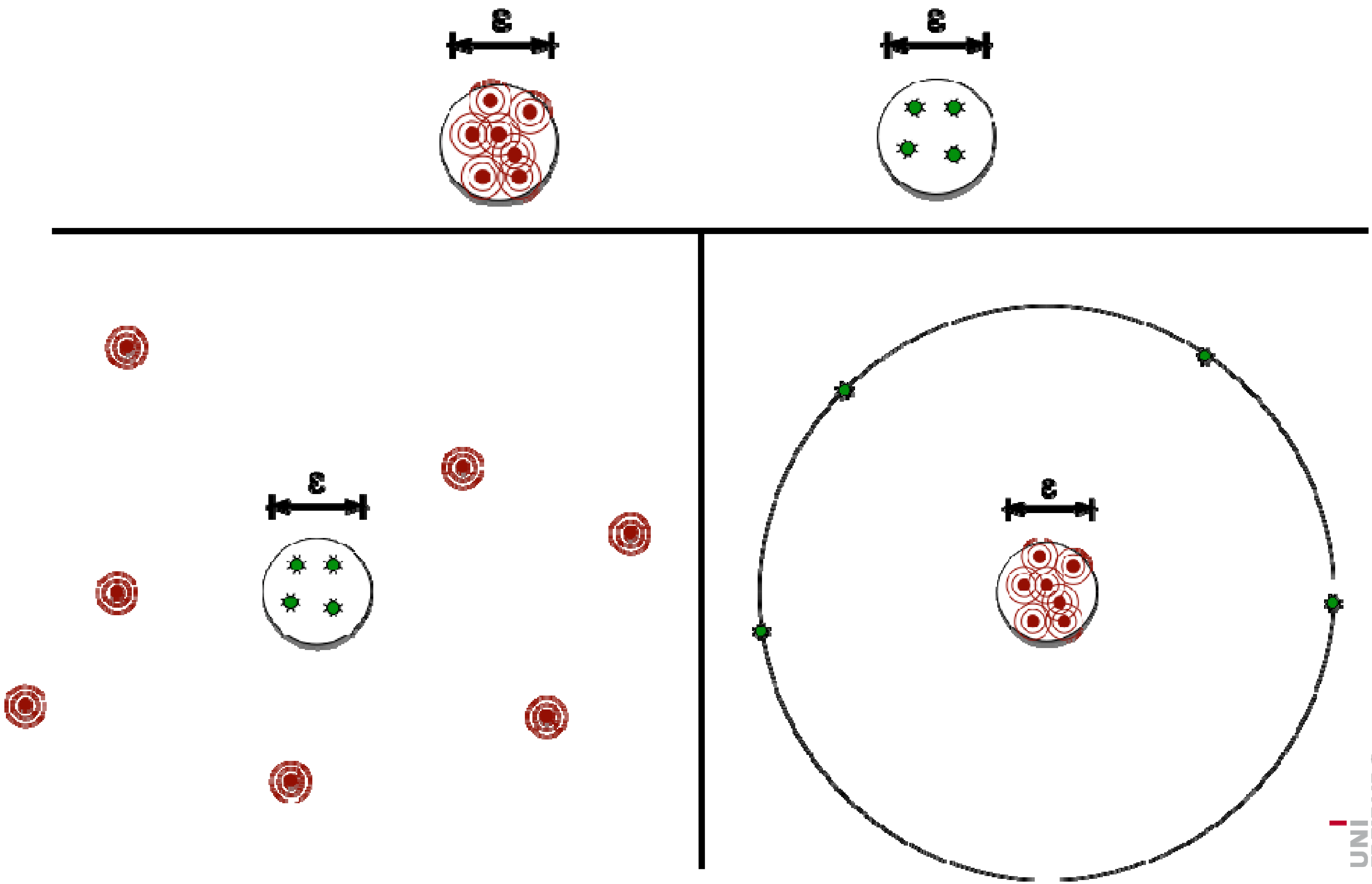


- $O(\log(1/\varepsilon))$, instead of $O(\varepsilon^{-5})$

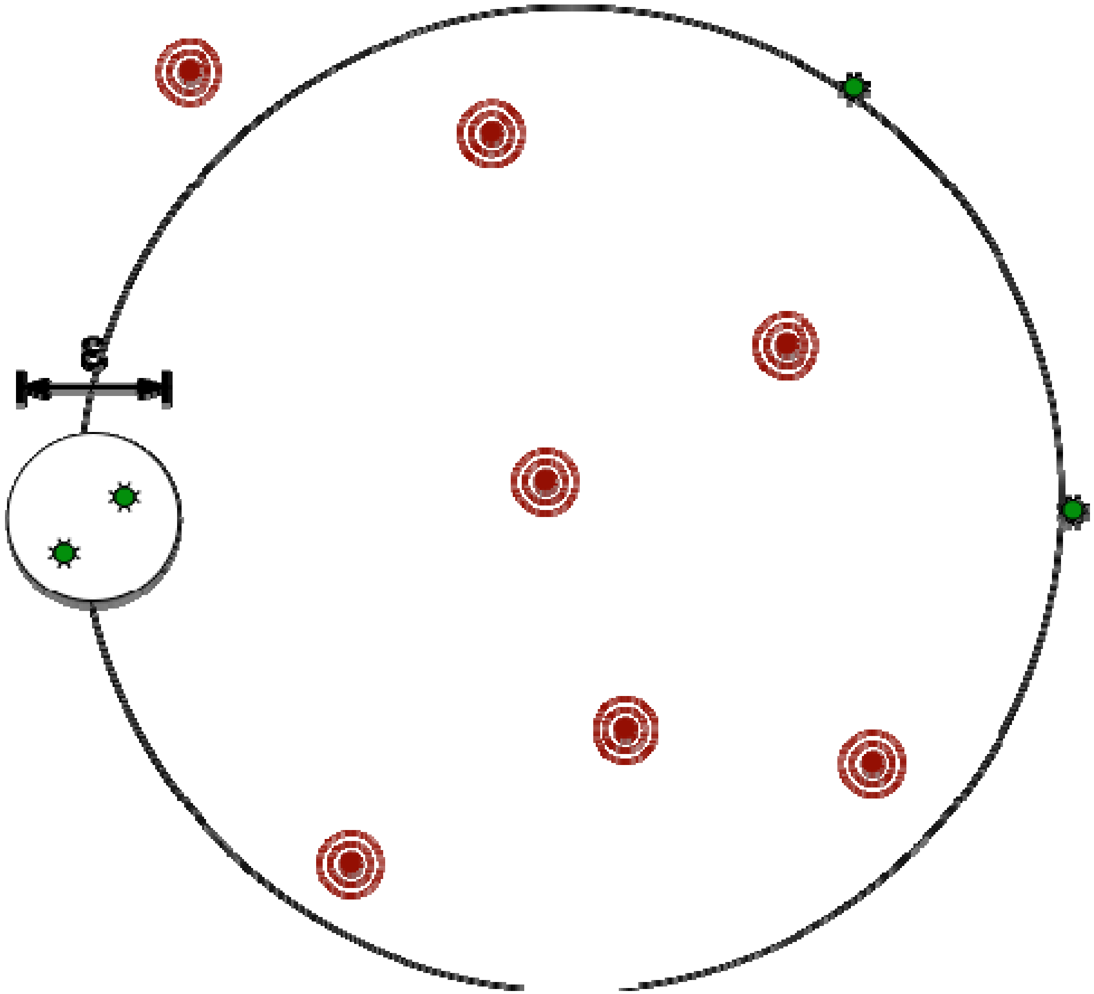
Properties of the Tree Search

- In the worst case each level of the tree has at most $O((D/\varepsilon)^5)$ nodes for four receivers
- Observation
  - If the minimum distances between receivers are large, then the number of nodes in each level in the tree is a constant
    - if the sound sources are “nicely” located
- There are some worst-case configurations which always lead to the maximum set of solutions, e.g.
  - all receivers in an ε-environment
  - all signal sources in an ε-environment
  - “camp fire” effect

Indistinguishable Worst Cases



Another Bad Case



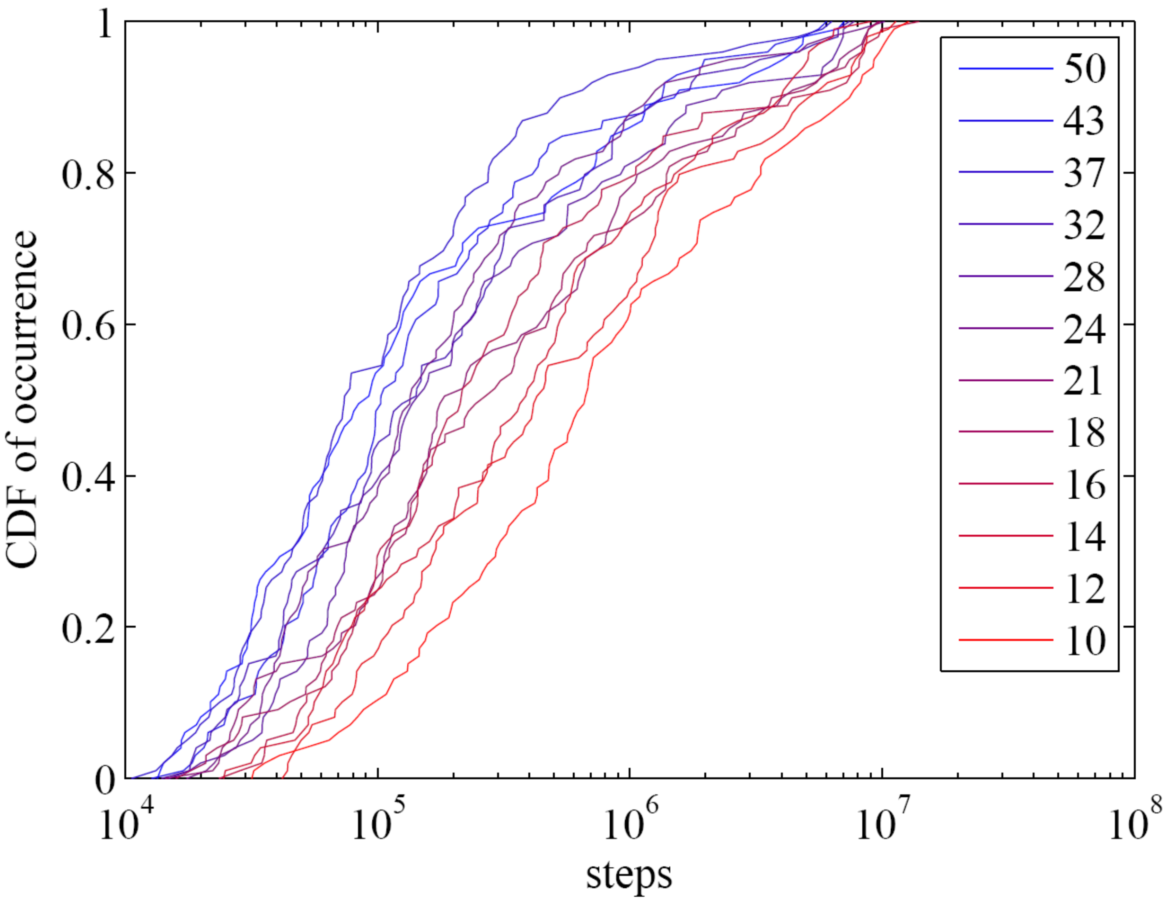
Conclusions

- First Complete Solution
  - For given ε>0 we can compute a set of receiver positions for which a set of signals satisfying the ε-Test exists in time $\mathcal{O}((\sqrt{2}/\epsilon)^{2n-3}mn^2)$.
- All solutions
  - If the maximum distance D of all receivers is known, then we can compute all different receiver positions satisfying the ε-Test.
  - The worst case run-time is $O((D/\varepsilon)^5)$
- Fast algorithm (conjecture)
  - If the ratio of maximum and minimum distances of receivers is constant and the signal positions are benevolent, then the worst-case run-time is O(-ln ε).

Thank you for your attention.

Experimental evaluation

- Number of test steps (tree size)



Experimental evaluation

- Minimum distance between receivers vs. size of search tree